Kinematic Analysis of Five-Bar Mechanism in Industrial Robotics

Louis Vathan.B, Hoshila Kumar and Brighton Issac John.H

Abstract---*This paper attempts to explain an innovative analysis for controlling planar five-bar chain mechanism. It also indulges with the implementing techniques of the mechanism in industrial robots.*

The previous analysis papers are studied and based on the basic equations for controlling the end-effector a new set of equations are made that can give the independent angles of the links directly. These angles are found to be useful in the real time controlling of the mechanism's end-effector position which is required in modern industrial robots.

The second set of analysis is the relation between the velocity of end-effector to speed of the individual link servos. The velocity control is obtained by simplifying the jacobian matrix of the angular velocities of the driving links. These angular velocities can be converted into individual speeds of the servos which act as the real time input. This kind of effective velocity control is suitable of path following end-effector industrial needs.

Keywords--- *End Effector, Five-Bar Manipulative System, Mechanism, Parallel Robot and Positioning Device.*

I. INTRODUCTION

RECENTLYas the rapid development of parallel robotics and controllable mechanism, the five-bar planar mechanism is widely used in mechanical design. The five link planar manipulative system (MS), shown in Figure 1, contains only rotational joints[1].

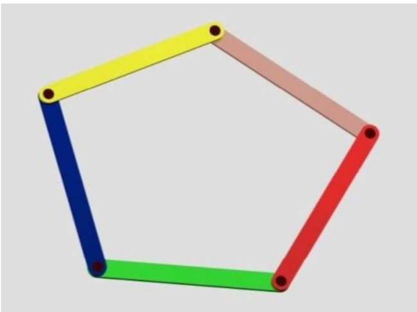

Figure 1: Model of Five-Bar MS

UG Student, Department of Mechanical Engineering, Sardar Raja College of Engineering, Alangulam, Tirunelveli, Tamil Nadu. E-mail:louisvathan@gmail.com

UG Student, Department of Mechanical Engineering, Sardar Raja College of Engineering, Alangulam, Tirunelveli, Tamil Nadu. E-mail:hoshilamech@gmail.com

Associate Professor, Department of Mechanical Engineering, Sardar Raja College of Engineering, Alangulam, Tirunelveli, Tamil Nadu. E-mail:engineer7077@yahoo.in

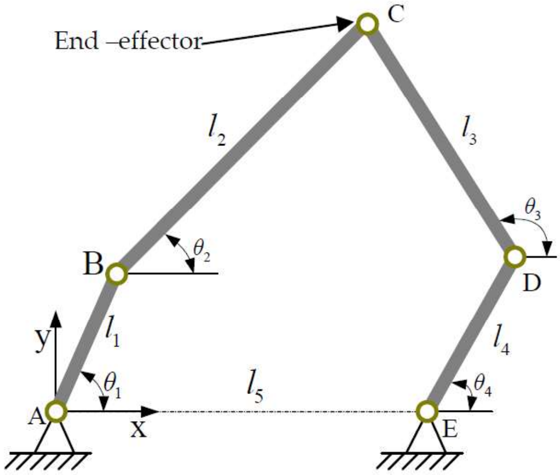

Figure 2: Schematic Diagram of Five-Bar Manipulative System [1]

Some parts are passive and others are active. The body 5 (l_5) is more particular as it stays immobile (it represents support) as shown in Figure 2. The bodies 1 and 4 are the driving bodies. With the help of the appropriate rotation of the actuating bodies, the characteristic point C of the MS can follow the desired planar trajectory in the borders of the working zone.

This paper assumes that the lengths of the links are l_1, l_2, l_3, l_4, and l_5, the angle between the links are$\theta_1, \theta_2, \theta_3 and \theta_4$. The additional angles $\theta_6 and \theta_7$ are used for kinematic analysis which can be derived from the link angles.

The five-bar manipulative system has the advantages of high efficiency, high payload and application flexibility. Five-bar MS can be found in many industrial applications as positing devices which improve the positional resolution, stiffness and force control of the manipulators.

II. POSITIONAL ANALYSIS

The main objective of the dynamic analysis of a five-bar MS is to derive the direct equations of the actuating angles from the end-effector coordinate equations. Initial position of the links can be obtained by the kinematic analysis of MS. According to the geometric relations of mechanism, as depicted in Figure 1, it can be derived the coordinates of point C: [1]

$$x_c = l_1 \cos\theta_1 + l_2 \cos\theta_2 = l_5 + l_4 \cos\theta_4 + l_3 \cos\theta_3 \ (1)$$

$$y_c = l_1 \sin\theta_1 + l_2 \sin\theta_2 = l_3 \sin\theta_3 + l_4 \sin\theta_4 \ (2)[1]$$

From (1) and (2), it can be found that $\theta_1$and $\theta_4$are independent in the system, and $\theta_2$and $\theta_3$can bedetermined by $\theta_1$and $\theta_4$as follows: [1]

$$\theta_3 = 2\tan^{-1}\left[\frac{\mathrm{A} \pm \sqrt{\mathrm{A}^2 + \mathrm{B}^2 - \mathrm{C}^2}}{\mathrm{B} - \mathrm{C}}\right] (3)$$

where,

$$A = 2l_3l_4\sin\theta_4 - 2l_1l_3\cos\theta_1$$

$$B = 2l_3l_5 - 2l_1l_3\cos\theta_1 + 2l_3l_4\cos\theta_4$$

$$C = l_1{}^2 - l_2{}^2 + l_3{}^2 + l_4{}^2 + l_5{}^2 - l_1l_4\sin\theta_1\sin\theta_4 - 2l_1l_5\cos\theta_1 + 2l_4l_5\cos\theta_4 - 2l_1l_4\cos\theta_1\cos\theta_4$$

From (2) and (3) above, we get,

$$\theta_2 = \sin^{-1}\left[\frac{l_3\sin\theta_3 + l_4\sin\theta_4 - l_1\sin\theta_1}{l_2}\right] (4)$$

Equations (3) and (4) gives an indirect way to determine the actuating angles in the manipulative system for a particular coordinates of the end-effector. This kind of equations needs simulation through SimMechanics or MATLAB software.

This kind of approach doesn't seem to be perfect for industrial robots which need an effective and dynamic control over the end-effector. For effective control some modifications are made over the pre-existing equations. These modified equations give the direct relation between the coordinates of the end-effector and link lengths to the actuating anglesθ_1 *and* θ_4.

By combining the equations (1) and (2) and eliminating the secondary angles $\theta_3$and θ_2 the following equations are obtained.

$$\theta_1 = 2\tan^{-1}\left[\frac{-B \pm \sqrt{\mathrm{A}^2 + \mathrm{B}^2 - \mathrm{C}^2}}{-\mathrm{A} - \mathrm{C}}\right] (5)$$

where,

$$A = x_c$$

$$B = y_c$$

$$C = \frac{l_1{}^2 - l_2{}^2 + x_c{}^2 + y_c{}^2}{2 * l_1}$$

$$\theta_4 = 2\tan^{-1}\left[\frac{-B \pm \sqrt{\mathrm{A}^2 + \mathrm{B}^2 - \mathrm{C}^2}}{-\mathrm{A} - \mathrm{C}}\right] (6)$$

where,

$$A = x_c - l_5$$

$$B = y_c$$

$$C = \frac{l_4{}^2 + l_5{}^2 - l_3{}^2 - 2x_cl_5 + x_c{}^2 + y_c{}^2}{2 * l_4}$$

The equations (5) and (6) give a direct control over the actuating angles without knowing the dependent angles θ_2 *and* θ_3. Since the links lengths are constant for a predefined robot the above equations can be simplified indeed. Then the coordinates of the end-effector (x_c *and* y_c) is the only inputs required for controlling the mechanism. This makes the semi-automated and automated control of the mechanism much easier than the traditional approach.

However the dependent and additional angles are also derived for future references in kinematic analysis is shown below:

$$\theta_2 = \sin^{-1}\left[\frac{y_c - l_1\sin\theta_1}{l_2}\right]$$

$$\theta_3 = \cos^{-1}\left[\frac{x_c - l_5 - l_4\cos\theta_4}{l_5}\right]$$

$$\theta_5 = 180 - \theta_4$$

$$\theta_6 = (\theta_1 - \theta_2)$$

$$\theta_7 = (\theta_3 - \theta_4)$$

Equations (5) and (6) are verified by applying sample link lengths and end-effector coordinates in the parametric software CREO and found to be correct.

The Figure 3 shows the placement of the above mentioned link angles in the five-bar manipulative system.

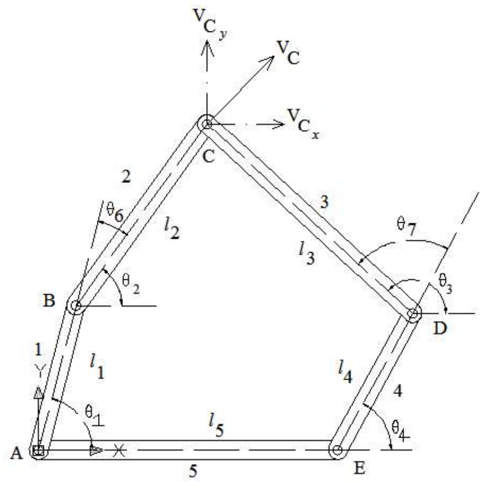

Figure 3: Auto CAD Generated Five-bar Manipulative System

III. Kinematic Analysis

The velocity $V = \left[V_{C_x}, V_{C_y}\right]^T$ of the characteristic point C is determined through the angular velocities $\dot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_4]^T$ of the bodies 1 and 4 and depends on transfer function of the mechanism.

Usually the transfer function is described by the Jacoby matrix J.

$$V = J\dot{\theta}(7)$$

This expression is known as forward kinematics problem and for the considered MS could be solved by using different approaches. The analytic symbolic solution could be particularly useful for making several conclusions concerning the singular configurations of the MS as well as MS metric. [2]

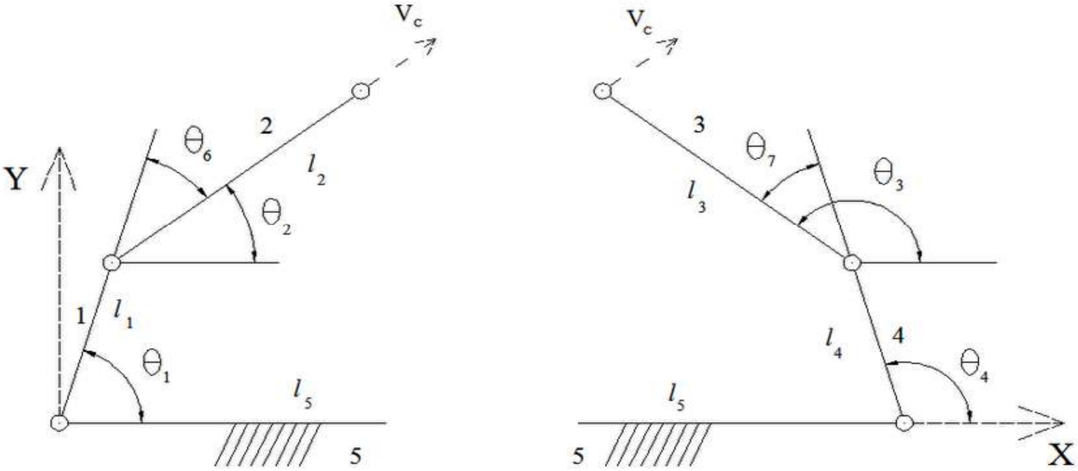

Figure 4: Representation of MS with Two Open Structures

The classical approach for solving such kind of problems requires the solution of standard position task (forward kinematics)$f(\theta_i) = X;\ i = 1{,}4;\ X = V = [x_c \quad y_c]^T$ or of the inverse kinematics.

After the obtained results are differentiated with respect to the general coordinates $\theta = [\theta_1, \theta_4]^T$. The forward kinematics (standard position task) has two solutions, the inverse-four. These arguments determine the necessity to search for other approaches for the analytical solution of the forward kinematics (position task).[2]

The Jacoby Matrix:

It is assumed that the MS is divided into two pairs as shown in Figure 4 representing two open planar kinematics chain with two links.

The matrix of Jacoby $J_{1,2}$ for each of them is known. For the left (J_1) MS we obtain,

$$J_1 = \begin{bmatrix} -A_{11} & -A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (8)$$

where:

$$A_{11} = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_6)$$
$$A_{12} = l_2 \sin(\theta_1 + \theta_6)$$
$$A_{21} = l_2 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_6)$$
$$A_{22} = l_2 \cos(\theta_1 + \theta_6)$$

Analogously we can obtain for the right system:

$$J_2 = \begin{bmatrix} -B_{11} & -B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (9)$$

where,

$$B_{11} = l_4 \sin\theta_4 + l_3 \sin(\theta_4 + \theta_7)$$
$$B_{12} = l_4 \sin(\theta_4 + \theta_7)$$
$$B_{21} = l_4 \cos\theta_4 + l_3 \cos(\theta_4 + \theta_7)$$
$$B_{22} = l_3 \cos(\theta_4 + \theta_7)$$

If it is admitted that the distance between both systems is l_1, and that they reach one and the same point B, and also the velocity V of that point B reached by the first and second MS is the same, we obtain the system,

$$\left.\begin{aligned} V_{C_x} &= -A_{11}\dot{\theta}_1 - A_{12}\dot{\theta}_6 \\ V_{C_y} &= A_{21}\dot{\theta}_1 + A_{22}\dot{\theta}_6 \\ V_{C_x} &= -B_{11}\dot{\theta}_4 - B_{12}\dot{\theta}_7 \\ V_{C_y} &= B_{21}\dot{\theta}_4 + B_{22}\dot{\theta}_7 \end{aligned}\right| \quad (10)$$

or in matrix form:

$$\begin{bmatrix} V \\ V \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{1,6} \\ \dot{\theta}_{4,7} \end{bmatrix}$$

where, $\dot{\theta}_{1,6} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_6 \end{bmatrix}$ and $\dot{\theta}_{4,7} = \begin{bmatrix} \dot{\theta}_4 \\ \dot{\theta}_7 \end{bmatrix}$

Eliminating the angular velocities $\dot{\theta}_6\ and\ \dot{\theta}_7$ in the passive joints for the forward kinematics problem it is obtained that,

$$\left.\begin{aligned} V_{C_x} &= -C_{11}\dot{\theta}_1 - C_{12}\dot{\theta}_4 \\ V_{C_y} &= C_{21}\dot{\theta}_1 + C_{22}\dot{\theta}_3 \end{aligned}\right| \quad (11)$$

or in a matrix form,

$$V_C = J\dot{\theta}$$

$$J = \begin{bmatrix} -C_{11} & -C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (12)$$

where,

$$C_{11} = A_{11} + A_{21}\frac{(B_{12}A_{21} - A_{11}B_{22})}{(A_{12}B_{22} - B_{12}A_{22})}$$

$$C_{12} = A_{12}\frac{(B_{11}B_{22} - B_{21}B_{12})}{(A_{12}B_{22} - B_{12}A_{22})}$$

$$C_{21} = A_{21} + A_{22}\frac{(B_{12}A_{21} - A_{11}B_{22})}{(A_{12}B_{22} - B_{12}A_{22})}$$

$$C_{22} = A_{22}\frac{(B_{11}B_{22} - B_{21}B_{12})}{(A_{12}B_{22} - B_{12}A_{22})} \qquad [2]$$

The above equations (8), (9), (10), (11) and (12), gives way to find the angular velocity of joints A, B, C and D and the linear velocity of the end-effector C. But these are complex which requires more resources to solve and find the active links angular velocities $\dot{\theta}_1 and \dot{\theta}_4$.

From these equations it is modified into simpler formulae which can directly give the values of active link's angular velocities.

$$\dot{\theta}_1 = \frac{\left(C_{22}V_{B_x}+C_{12}V_{B_y}\right)}{(C_{11}C_{21}-C_{11}C_{22})} (13)$$

$$\dot{\theta}_4 = \frac{\left(C_{21}V_{B_x}+C_{11}V_{B_y}\right)}{(C_{11}C_{22}-C_{12}C_{21})} (14)$$

Here the inputs are assumed to be the velocity required at the end-effector, and the constants which values are given above. The velocity of the end-effector is differentiated into an x and y components $(V_{B_x} and V_{B_y})$ based on the direction of movement. These components are needed to be found out before applying in the equations (13) and (14).

The speed of the individual motors 1 and 4 controlling the links A and B can be found out easily using the following relations:

$$N_1 = \frac{(60*\dot{\theta}_1)}{(2*\pi)} (15)$$

$$N_4 = \frac{(60*\dot{\theta}_4)}{(2*\pi)} (16)$$

For path movements of the end-effector the speed and angles can be simultaneously varied using the proposed equations.

From these modified equations the real time variable speed can be achieved at the end-effector by changing the speed of the controller motors.

IV. IMPLEMENTATION

This five-bar MS relates generally to robotic systems of the type which are employed in automation, and more particularly, to a robotic system which provides a measure of flexibility intermediate of cam-driven or linkage-based conventional mechanisms, where serial robots which provide great flexibility, but at high cost.

There is a need to provide a robotic system which is useful in automation and achieves a reasonable measure of flexibility at relatively low cost. The term "flexibility" refers to the ability of a robotic system to be deployable to perform a variety of tasks. In this context, the term "task" refers to a mechanized motion in one, two, or three dimensional space along a prescribed path.[4]

Serial industrial robots are widely employed in automated manufacturing operations, such as in the assembly of mechanical and electronic components, welding, painting, sealing, etc..flexible manufacturing applications demand the use of such expensive robots.[4]

The need for only limited flexibility arises from the fact that in most manufacturing environments, there will always be fundamental similarities between the different products on a given production line. Such similarities include, for example, overall size, overall weight, desired mechanical paths, etc. Consequently, there are present corresponding similarities in the robot workspace requirements, the paths traced by the end-effector, the accuracy and precision with which the task must be accomplished, and the operating speed.[4]

Therefore the proposed mechanism provides an inexpensive robotic system which exhibits greater flexibility than conventional hard automation systems.

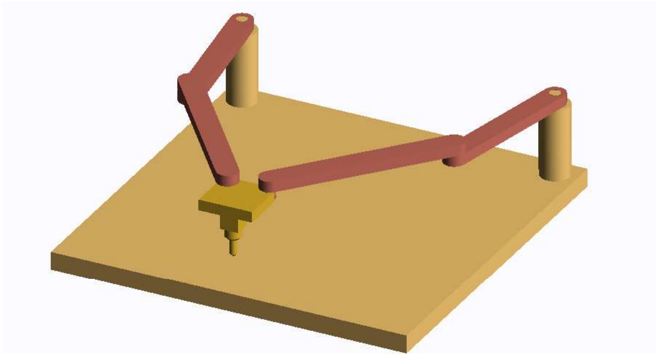

Figure 5: Featured Automated Drilling Machine using Five-Bar MS

The Figure 5 shows a CREO modeled automated robot which can make drills in any of the indicated coordinates based on the requirements.

Some of the applications of five-bar mechanism in robotics are listed below.

- In high speed, high-accuracy positioning with limited workspace, such as in assembly of PCBs.
- As micro manipulators mounted on the end-effector of larger but slower serial manipulators.
- As high speed/high-precision milling machines.
- As high speed positioning device in automated drilling machine.
- In precision surface finish measuring machines.
- As path tracker in automated welding machine.
- As commercial pick and place robot.
- For generating walking gait in biped robots and swimming gait in swimming robots.
- In automatic planar measuring devices.

Many of these robotic applications need a path to be generated by the end-effector for operation. For this the trajectory control using numerical methods can be made.

Trajectory Control of a Welding Robot:

It is assumed that an automated welding to be made repeatedly over several work pieces in a manufacturing unit. The traditional robots for this kind are serial robots. But the five-bar manipulative system gives an inexpensive solution for that.

Let the trajectory path is a straight line of 60mm with the slope of 45° and the link lengths are assumed to be l_1=80mm, l_2=40mm, l_3=50mm, l_4=50mm and l_5=40mm.

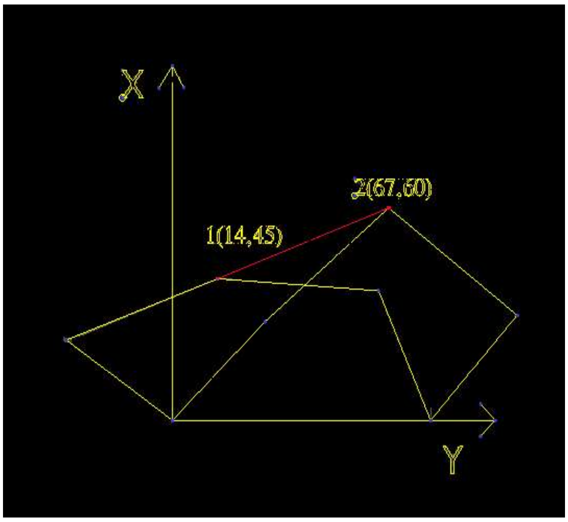

Figure 6: Trajectory Motion of End-Effector in Five-Bar Mechanism

The end coordinates of the trajectory be 1(67, 60) and 2(14, 45) with a slope of 0.386 as in Figure 6.

So the straight line equation is y_c=0.386x_c.

Here the equations can be further simplified by eliminating y_c in terms of x_c.

The trajectory control can be achieved using the numerical methods as shown below:

$$y_c = \{y_{c_1}, y_{c_2}, y_{c_3}, \dots\dots\dots, y_{c_n}\} \quad (17)$$

From the obtained coordinates of the trajectory the driving angles and their angular velocities can be obtained.

$$\theta_1 = \{\theta_{1_1}, \theta_{1_2}, \theta_{1_3}, \dots\dots\dots, \theta_{1_n}\} \quad (18)$$

$$\theta_4 = \{\theta_{4_1}, \theta_{4_2}, \theta_{4_3}, \dots\dots\dots, \theta_{4_n}\} \quad (19)$$

The velocity of the end-effector is determined based on the welding speed required and consequently the angular velocities of the links can be found out and tabulated as follows:

$$\dot{\theta}_1 = \{\dot{\theta}_{1_1}\dot{\theta}_{1_2}\dot{\theta}_{1_3}, \dots\dots\dots, \dot{\theta}_{1_n}\} \quad (20)$$

$$\dot{\theta}_4 = \{\dot{\theta}_{4_1}\dot{\theta}_{4_2}\dot{\theta}_{4_3}, \dots\dots\dots, \dot{\theta}_{4_n}\} \quad (21)$$

From these angular velocity values the speed of the servos can be found out for every point of the trajectory.

V. Effect of Proposed Control System Comparing with the Traditional

The closed loop mechanism structure was traditionally controlled based on the hybrid systems. Greennough et al (1995) developed a system which has a Constant Velocity (CV) motor coupled with a flywheel and a servomotor for controlling the five bar mechanism. The CV motor with flywheel provides the required torque and the servomotor takes care of the trajectory control of the mechanism. This method is considered as economic because of lower cost of the CV motor.

But the error tracking is the major criteria in these systems because of the deviations in the desired velocity in the CV motor. This creates deviation in the achievement of the desired coordinates by the mechanical system which is generally corrected by the adaptive control methodology. For such control systems a lot of computational resources are to be put up for controlling the mechanism.

The simplified analysis results presented in this work are capable of increasing the computational efficiency of the mechanism. It also increases the accuracy of the trajectory tracking of the mechanism.

Since the motor technology have been improved a lot in the last decade, the servo motors are now produced with high ratio of payload to the motor power capacity and with high motion precision at high-speed operations. It is also proved that the servo motor driven mechanisms can operate in a wide range of motion and have an extreme acceleration profile.

So it is obvious to use two servomotors for controlling the mechanism rather than using hybrid machine control system. For example, the Mecademic Inc. have successfully developed a parallel robot called DexTAR (Dextrous twin Arm Robot) which is a five bar mechanism for educational purposes. It was driven by two servomotors posses a resolution of 0.011^o and a maximum linear speed of 300mm/s.

Figure 7: DexTAR (Dextrous Twin-Arm Robot) - Three-Axis Dual-Arm SCARA Robot for Education and Training[6]

The equations (5), (6), (13) and (14) can be directly included in the program for effective control of the mechanism with lesser computational resources.

The above stated equations have the capability to replace the traditional standard equations for controlling the mechanism with much fewer errors in the generation of end-effector coordinates. However the proposed control system also requires error tracking methodology but the errors would be comparatively low.

VI. Conclusion

The numerical model of a five bar mechanism is developed in this work. The proposed numerical model is found to be simplified one on comparing with the previous analysis reports made by various researchers. It has been observed that the equations predicted in this work give a direct relation between the coordinates of the end-effector and the angles of the driving bodies 1 and 4.

The five bar mechanism is stiffer, faster and more accurate than the serial robots, but the complexity of the control makes it harder to use. This complexity of controlling can be reduced with the methodology predicted in this paper. The trajectory control of the mechanism based on the numerical method is also explained in this paper. The comparison between the traditional and current control method is highlighted with examples. Thus the proposed equations can be the optimized solution for controlling the five bar mechanism.

REFERENCES

[1] Bin Zi, Jianbin Cao and Zhencai Zhu, *"Dynamic Simulation of Hybrid-driven Planar Five-bar Parallel Mechanism Based on SimMechanics and Tracking Control,"* Int J Adv Robotic Sy, 2011, Vol. 8, No. 4, 28-33.

[2] Ivan Chavdarov *"Kinematics and Force Analysis of a Five-Link Mechanism by the Four Space Jacoby Matrix"* Proceedings to the 55th Conference of Problems of Engineering Cybernetics and Robotics 2005, 53 – 63.

[3] Chuan Li, Theingi, I-Ming Chen, *"Cases Study on the Singularity of Five-Bar Planar Mechanism with Differential Gear Drives,"* Proceeding of the 6th International Conference on Engineering Design & Automation, Maui, Hawaii, 2002, 399 – 404.

[4] Sridhar Kota, Ann Arbor, Mich, "*Adjustable Robotic Mechanism*," United States Patent, Apr. 28, 1992, USOO5107719A.

[5] Reza Moazed, "*Experimental Study of a Two DOF Five Bar Closed-Loop Mechanism*", The Department of Mechanical Engineering, University of Saskatchewan, Canada, August 2006.

[6] "*Dextar – Dual-arm SCARA Robot for Education and Training*", http://www.mecademic.com/dextar.html, Mecademic Inc., 2014, Web.

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Louis Vathan.B was born on 28th May, 1994 at Tenkasi, Tamil Nadu, India. Presently, he is doing his Final Year Under Graduate Engineering Degree in Sardar Raja College of Engineering, Tirunelveli. His interest is on Optimized Design Methodology of Mechanisms for Robotic Applications. (E-Mail: louisvathan@gmail.com)

Hoshila Kumar has born on 5th May, 1992 at Bihar, India. He has completed Diploma in Mechanical Engineering in S.A. Raja Polytechnic College, Vadakangulam in 2012. At Present, he is doing his final year Under Graduate Engineering Degree in Sardar Raja College of Engineering, Tirunelveli. His interest is on Design and Simulation of Effective End-Effector Control. (E-Mail: hoshilamech@gmail .com)

Brighton Isaac John. H was born on 22nd August 1979 at Palayamkottai, Tirunelveli District, Tamil Nadu State, India. He received his B.E degree in Mechanical Engineering in 2000 and M.Tech degree in Manufacturing Engineering in 2001 from Manonmaniam Sundaranar University, Tirunelveli. He had his Ph.D degree in the Faculty of Mechanical Engineering in the area of "Elasto-plastic behavior of structures having defects" from Anna university in 2013.He is working as a Professor in Mechanical Engineering Department of Sardar Raja College of Engineering, Alangulam, Tirunelveli District. He has published his work in seven International Journals. He has presented his research findings in one national and two International Conferences. He has attended many faculty training programs. His area of interest includes Finite element analysis, Fracture mechanics and Engineering design. (E-Mail: engineer7077@yahoo.in)